30-11-20

• Επανάληψη των προηγούμενων (θεωρήματα 12, 13, 14, 15)

• **Μη ομογενείς γραμμικές εξισώσεις με σταθερούς συντελεστές.**

$a_n y^{(n)}(t) + a_{n-1} y^{(n-1)}(t) + \dots + a_1 y'(t) + a_0 y(t) = b(t),\ t \in I,\ a_i \in \mathbb{R}\ (i=0,\dots,n),\ b \in C(I)$

(I) Η μέθοδος των αγνώστων σταθερών.

• $b(t) = P_m(t)$, $a_0,\dots,a_{k-1} = 0 \rightarrow y_\mu^{(t)} = p_m(t)$

Πχ Παράδειγμα 3i: $y^{(4)} + y'' = t^3 + 1$.

$\rightarrow y_\mu'' = at^3 + bt^2 + ct + d$

$\rightarrow y_\mu'' = t^3 - 6t + 1$

$\rightarrow y_\mu = \frac{1}{20} t^5 - t^3 + \frac{1}{2} t^2$

• $b(t) = e^{kt} P(t), \rightarrow y_\mu(t) = z(t) e^{kt}$

Πχ Παράδειγμα 3iii: $y''' + y'' + 2y = t^2 e^{-2t}$

$y_\mu = z e^{-2t}$

$\Rightarrow z''' - 5z'' + 8z' - 2z = t^2$

$\Rightarrow z_\mu = at^2 + bt + c$

$\Rightarrow z_\mu(t) = -\frac{1}{2} t^2 - 4t - \frac{27}{2}$

• $b(t) = P(t) e^{\sigma t} \cos(\tau t), \sin(\tau t), \rightarrow y_\mu(t) = z(t) e^{(\sigma + i\tau)t}$, $\mathrm{Re}\, z, \mathrm{Im}\, z$

Πχ Παράδειγμα 3iv,v: $y'' - 2y' + y = t e^{-t} \sin(2t), \cos(2t)$

$\Rightarrow y'' - 2y' + y = t e^{-t+i2t}$

$\Rightarrow y_\mu = z e^{-t+i2t}$

$\Rightarrow z'' + 4(-1+i)z' - 8iz = t$

$\Rightarrow z_\mu(t) = bt + c$

$\Rightarrow z_\mu = \frac{1}{8}it + \frac{1}{16}(-1+i)$

$\Rightarrow y_\mu(t) = z_\mu(t)e^{-t+i2t} = e^{-t}[\cos 2t + i\sin 2t]\left[\frac{1}{8}it + \frac{1}{16}(-1+i)\right].$

$\Rightarrow \operatorname{Re} y_\mu(t) = e^{-t}\left[\frac{-1}{16}\cos(2t) - \frac{1}{8}\left(t+\frac{1}{2}\right)\sin(2t)\right].$

Πχ **Άσκηση B-43:** $y'' - 2by' + cy = 0$, b, c σταθερές, y: λύση

$y(0) = y(1) = 0 \to y(n) = 0,\ n \in \mathbb{Z}$

<u>Λύση</u>: Χαρακτηριστικό πολυώνυμο: $\lambda^2 - 2b\lambda + c = 0$

Ρίζες: $\lambda_{1,2} = 2b \pm \sqrt{4b^2 - 4c} \Rightarrow \lambda_{1,2} = b \pm \sqrt{b^2 - c}$

(i) $b^2 > c$:

$y_1 = e^{\lambda_1 t}$, $y_2 = e^{\lambda_2 t}$, $\lambda_1 \neq \lambda_2$, $\lambda_1, \lambda_2 \in \mathbb{R}$

$y(t) = c_1 y_1(t) + c_2 y_2(t) = c_1 e^{\lambda_1 t} + c_2 e^{\lambda_2 t}$

$$\left.\begin{array}{l} y_1(0) = 0 \to c_1 + c_2 = 0 \\ y_1(1) = 0 \to c_1 e^{\lambda_1} + c_2 e^{\lambda_2} = 0 \end{array}\right\} \Rightarrow c_1 = c_2 = 0$$

(ii) $b^2 = c$

$\to y(t) = c_1 e^{\lambda t} + t e^{\lambda t}$, $\lambda = b$

$\Rightarrow c_1 = c_2 = 0$

(iii) $b^2 < c$

$\delta = \sqrt{c - b^2}$

$\lambda_{1,2} = b \pm i\delta$

$y(t) = c_1 e^{bt}\cos(\delta t) + c_2 e^{bt}\sin(\delta t)$

$y(0) = 0 \Rightarrow c_1 + c_2 e^0 \sin(0) = 0 \Rightarrow c_1 = 0$

$y(1) = 0 \Rightarrow c_2 e^b \sin(\delta) = 0 \Rightarrow \delta = k\pi.$

$\to y(t) = c_2 e^{bt}\sin(k\pi t) = y_2$

$t \in \mathbb{R} \Rightarrow y_2(t) = 0$